Día Mes Año

Universidad Central Del Ecuador

Facultad De Filosofía, Letras y Ciencias De La Educación

Pedagogía De Las Ciencias Experimentales Informática

Trabajo Grupal

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* 15 ejercicios de ecuaciones diferenciales de variables separables.

① $\frac{dy}{dx} = 2xy$

$\frac{dy}{y} = 2x\,dx$

$\int \frac{dy}{y} = \int 2x\,dx$

$\ln|y| = x^2 + C$

$y = Ce^{x^2}$ //

② $\frac{dy}{dx} = \frac{x}{y}$

$x\,dy = x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C$

$y^2 - x^2 = C$ //

③ $\frac{dy}{dx} = 3x^2y$

$\frac{dy}{y} = 3x^2\,dx$

$\int \frac{dy}{y} = \int 3x^2\,dx$

$\ln|y| = x^2 + C$

$y = Ce^{x^2}$ //

④ $\frac{dy}{dx} = xy^2$

$\frac{dy}{y^2} = x\,dx$

$\int y^{-2}\,dy = \int x\,dx$

$\frac{-1}{y} = \frac{x^2}{2} + C$

$y = -\frac{2}{x^2 + C}$ //

⑤ $\frac{dy}{dx}\ e^{2x}y$

$\frac{dy}{y} = e^{2x}\,dx$

$\int \frac{dy}{y} = \int e^{2x}\,dx$

$\ln|y| = \frac{e^{2x}}{2} + C$

$y = Ce^{\frac{e^{2x}}{2}}$ //

⑥ $\frac{dy}{dx} = \frac{\cos x}{y}$

$y\,dy = \cos x\,dx$

$\int y\,dy = \int \cos x\,dx$

$\frac{y^2}{2} = \operatorname{sen} x + C$

$y^2 = 2\operatorname{sen} x + C$ //

⑦ $\frac{dy}{dx} = \frac{1}{y} \sec^2 x$

$y\,dy = \sec^2 x\,dx$

$\int y\,dy = \int \sec^2 x\,dx$

$\frac{y^2}{2} = \tan x + C$

$y^2 = 2\tan x + C_1$

⑧ $\frac{dy}{dx} = e^{x-y}$

$\frac{dy}{dx} = e^x e^{-y}$

$e^y\,dy = e^x\,dx$

$e^y = e^x + C$

$y = \ln(e^x + C)$

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9.- $\frac{dy}{dx} = \frac{x+1}{y-1}$

$(y-1)\,dy = (x+1)\,dx$

$\int (y-1)\,dy = \int (x+1)\,dx$

$\frac{(y-1)^2}{2} = \frac{(x+1)^2}{2} + C$

$(y-1)^2 = (x+1)^2 + C$ //

10.- $\frac{dy}{dx} = \frac{y^2}{x^2}$

$\frac{dy}{y^2} = \frac{dx}{x^2}$

$\int y^{-2}\,dy = \int x^{-2}\,dx$

$-\frac{1}{y} = -\frac{1}{x} + C$

$\frac{1}{y} = \frac{1}{x} - C$

$y = \frac{x}{1 - Cx}$ //

11.- $\frac{dy}{dx} = \frac{\ln x}{y}$

$y\,dy = \ln x\,dx$

$\int y\,dy = \int \ln x\,dx$

$\frac{y^2}{2} = x\ln x - x + C$

$y^2 = 2(x\ln x - x) + C$ //

12.- $\frac{dy}{dx} = y\tan x$

$\frac{dy}{y} = \tan x\,dx$

$$\int \frac{dy}{y} = \int \tan x \, dx$$

$$\ln|y| = -\ln|\cos x| + C$$

$$\ln|y| = \ln|\sec x| + C$$

$$y = C \sec x \,_{11}$$

13.- $\frac{dy}{dx} = \frac{e^x}{e^y}$

$$e^y dy = e^x dx$$

$$\int e^y dy = \int e^x dx$$

$$e^y = e^x + C$$

$$y = \ln(e^x + C) \,_{11}$$

14.- $\frac{dy}{dx} = \frac{2x}{3y^2}$

$$3y^2 dy = 2x \, dx$$

$$\int 3y^2 dy = \int 2x \, dx$$

$$y^3 = x^2 + C$$

$$y^3 = x^2 + C \,_{11}$$

15.- $\frac{dy}{dx} = \frac{x^2+1}{y^2+1}$

$$(y^2+1) dy = (x^2+1) dx$$

$$\int (y^2+1) dy = \int (x^2+1) dx$$

$$\frac{y^3}{3} + y = \frac{x^3}{3} + x + C$$

$$\frac{y^3}{3} + y = \frac{x^3}{3} + x + C \,_{11}$$

Ecuaciones Diferenciales Homogéneas

1. $\frac{dy}{dx} = \frac{x+2y}{2x-y}$

$y = vx \qquad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{1-v}{1+v}$

$\frac{1+v}{1-2v-v^2}\,dv = \frac{dx}{x}$

$-\frac{1}{2}\ln|1-2v-v^2| = \ln|x| + C$

$$= -\frac{1}{2}\ln\left|1-2\frac{y}{x}-\left(\frac{y}{x}\right)^2\right| = \ln|x| + C$$ //

2. $\frac{dy}{dx} = \frac{x+y}{x}$

$y = vx, \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = 1 + v$

$x\frac{dv}{dx} = 1$

$v = \ln|x| + C$

$$y = x(\ln|x| + C)$$ //

3. $\frac{dy}{dx} = \left(\frac{y}{x}\right)^2$

$v + x\frac{dv}{dx} = v^2$

$\frac{dv}{v(v-1)} = \frac{dx}{x}$

$\ln\left|\frac{v-1}{v}\right| = \ln|x| + C$

$$y = \frac{x}{1+Cx}$$ //

4. $\frac{dy}{dx} = \frac{x+2y}{2-x}$

$v + x\frac{dv}{dx} = \frac{1+2v}{2-v}$

$\frac{2-v}{1-v^2}dv = \frac{dx}{x}$

$-\frac{1}{2}\ln|v-1| - \frac{3}{2}\ln|v+1| = \ln|x| + C$

$\boxed{(v-1)(v+1)^3 = \frac{C}{x^2}, \quad v = \frac{y}{x}}$ //

5. $(x^2+y^2)dx - 2xy\,dy = 0$

$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

$v + x\frac{dv}{dx} = \frac{1+v^2}{2v}$

$\boxed{\frac{2v}{(v-1)^2}dv = -\frac{dx}{x}}$ //

6. $\frac{dy}{dx} = \frac{x+y}{x}$

$y = vx, \quad \frac{dy}{dx} = v + x\frac{dy}{dx}$

$v + x\frac{dv}{dx} = 1 + v$

$dv = \frac{dx}{x} \Rightarrow v = \ln|x| + C$

$\boxed{y = x(\ln|x| + C)}$ //

7. $\frac{dy}{dx} = \frac{x-y}{x}$

$y = vx, \quad \frac{dy}{dx} = v + x\frac{dy}{dx}$

$v + x\frac{dv}{dx} = 1 - v$

$\frac{dv}{1-2v} = \frac{dx}{x}$

$-\frac{1}{2}\ln|1-2v| = \ln|x| + C$

$\boxed{= -\frac{1}{2}\left|1 - 2\frac{y}{x}\right| = \ln|x| + C}$ //

1.- $x\frac{dy}{dx} = y + x$

$\frac{dy}{dx} = \frac{y}{x} + 1$

$x\frac{du}{dx} = 1 \Rightarrow du = \frac{dx}{x}$

$u = \ln|x| + C$

$\frac{y}{x} = \ln|x| + C \Rightarrow y = x(\ln|x| + C)$ //

2.- $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$

$\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x}$

$x\frac{du}{dx} + u = \frac{1}{u} + u$

$x\frac{du}{dx} = \frac{1}{u} \Rightarrow u\,du = \frac{dx}{x}$

$\frac{u^2}{2} = \ln|x| + C_1 \Rightarrow u^2 = 2\ln|x| + C$

$\frac{y^2}{x^2} = 2\ln|x| + C \Rightarrow y^2 = x^2(2\ln|x| + C)$ //

3.- $(x^2 - y^2)dx + 2xy\,dy = 0$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} =$

$x\frac{du}{dx} = -\frac{u}{2} - \frac{1}{2u} =$

$\frac{2u}{u^2+1}du = -\frac{dx}{x} \Rightarrow \ln(u^2+1) = -\ln|x| + C_1$

$\frac{y^2}{x^2} + 1 = \frac{C}{x} \Rightarrow \frac{x^2 + y^2}{x^2} = \frac{C}{x} \Rightarrow x^2 + y^2 = Cx$ //

4.- $\frac{dy}{dx} = \frac{x+y}{x-y}$

$x\frac{du}{dx} + u = \frac{1+u}{1-u}$

$\frac{x\,du}{dx} = \frac{1+u}{1-u} - u \Rightarrow x\frac{du}{dx} = \frac{1+u^2}{1-u}$

$\int \frac{1}{1+u^2}du - \int \frac{u}{1+u^2}du = \int \frac{dx}{x}$

$\arctan(u) - \frac{1}{2}\ln(1+u^2) = \ln|x| + C$

$\arctan\left(\frac{y}{x}\right) - \frac{1}{2}\ln(x^2 + y^2) = C_1$

5.- $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2+y^2}}{x} = \frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}}$$

$$x\frac{du}{dx} + u = u + \sqrt{1+u^2}$$

$$x\,\frac{du}{dx} = \sqrt{1+u^2} \Rightarrow \frac{du}{\sqrt{1+u^2}} = \frac{dx}{x}$$

$$\ln|u + \sqrt{1+u^2}| = \ln|x| + C_1$$

$$u + \sqrt{1+u^2} = Cx$$

$$\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}} = Cx \Rightarrow y + \sqrt{x^2+y^2} = Cx^2$$

6.- $(x^3+y^3)\,dx - 3xy^2\,dy = 0$

$$\frac{dy}{dx} = \frac{x^3+y^3}{3xy^2} = \frac{1}{3}\left(\frac{x^2}{y^2} + \frac{y}{x}\right)$$

$$x\frac{du}{dx} + u = \frac{1}{3u^2} + \frac{u}{3}$$

$$x\frac{du}{dx} = \frac{1}{3u^2} - \frac{2u}{3} = \frac{1-2u^3}{3u^2}$$

$$\frac{3u^2}{1-2u^3}\,du = \frac{dx}{x}$$

$$-\frac{1}{2}\ln|1-2u^3| = \ln|x| + C_1 \Rightarrow \ln|1-2u^3| = -2\ln|x| + C_2$$

$$1 - 2u^3 = \frac{C}{x^2}$$

$$1 - 2\frac{y^3}{x^3} = \frac{C}{x^2} \Rightarrow x^3 - 2y^3 = Cx$$

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7.- $\frac{dy}{dx} = \frac{2y - x}{y + x}$

$$\frac{dy}{dx} = \frac{2\left(\frac{y}{x}\right) - 1}{\frac{y}{x} + 1}$$

$$x\frac{du}{dx} + u = \frac{2u - 1}{u + 1}$$

$$x\frac{du}{dx} = \frac{2u - 1 - u(u+1)}{u+1} = \frac{-u^2 + u - 1}{u+1}$$

$$\frac{u+1}{u^2 - u + 1}\,du = -\frac{dx}{x}$$

$$\frac{1}{2}\ln(u^2 - u + 1) + \sqrt{3}\arctan\left(\frac{2u-1}{\sqrt{3}}\right) = -\ln|x| + C$$

$$\ln(y^2 - xy + x^2) + 2\sqrt{3}\arctan\left(\frac{2y - x}{x\sqrt{3}}\right) = C //$$

8.- $x^2\frac{dy}{dx} = y^2 + xy$

$$\frac{dy}{dx} = \left(\frac{y}{x}\right)^2 + \frac{y}{x}$$

$$x\frac{du}{dx} = u^2 \Rightarrow \frac{du}{u^2} = \frac{dx}{x}$$

$$-\frac{1}{u} = \ln|x| + C$$

$$-\frac{x}{y} = \ln|x| + C \Rightarrow y = \frac{-x}{\ln|x| + C} //$$